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ENROLLMENT FORECASTING WITH DOUBLE EXPONENTIAL SMOOTHING: TWO METHODS FOR OBJECTIVE WEIGHT FACTOR SELECTION

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Mary Corcoran
University of Minnesota
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Abstract

The merits of double exponential smoothing are discussed relative to other types of pattern-based enrollment forecasting methods. The basic assumptions and formulas for its use are outlined. The difficulties associated with selecting an appropriate weight factor are discussed, and their potential effects on prediction results is illustrated. Two methods for objectively selecting the "best" weight factor are described and analyzed, and evidence is presented suggesting they may be used effectively in the enrollment-forecasting process.

Introduction

A variety of classification systems for categorizing enrollment projection methods have been proposed. For example, see the writings of Lins (1960), Mangelson, Norris, Poulton, and Seeley (1974), and Wing (1974). Most forecasting methods can be categorized within four basic types: (1) Pattern based. Methods in this category attempt to identify an underlying pattern in historical data, describe it in mathematical terms, and then extrapolate it into the future. Specific examples range in complexity from simple averaging and ratio methods to cohort survival, Markov-chain and the various curve-fitting formulas. (2) Correlation analysis. This type of approach develops projections on the basis of the numerical values of "leading indicator" variables which have been demonstrated, e.g., through the use of multiple regression, to have a strong relationship to enrollment levels. Because an attempt is made to identify variables with a more or less direct impact on college enrollments, such as numbers of graduating high school seniors, methods of this type are often

referred to as "causal methods". (3) Intention surveys. Intention surveys involve sampling specific groups to determine college attendance plans. Although this method can be relatively costly, it has been shown to be useful and reliable in some situations, as demonstrated by Russell and Hoffman (1977). (4) Professional judgment. Although this is probably the most commonly used approach, it is perhaps the most under-rated. The lack of objectivity of this method may be more than offset by the benefit of experienced judgment and interpretation of subtle factors influencing enrollment not easily captured by mathematical models. Also, any or all of the other techniques may contribute to the educated prediction of an experienced administrator.

Double Exponential Smoothing

Among the various pattern based methods, double exponential smoothing has enjoyed success as a forecasting tool because it is easily understood, simple to apply, and, in so many situations, reasonably accurate. Furthermore, its ability to selectively weight the historical data points on the basis of assumed relative importance of more recent information gives this method a type of primitive "learning ability" with regard to the effects of random fluctuations on the underlying pattern. In this sense, it has significant advantages over other simple methods such as moving averages, as described in Parker, Gardner, and Matt (1979).

The single exponential smoothing formula upon which the double exponential method is based is as follows:

$$S'_{t+1} = \alpha X_t + (1 - \alpha) S'_t$$

Where S'_{t+1} = Single exponential forecasting for period t + 1

X_t = Actual value for period t

S'_t = Single forecast for period t

α = Weighting factor (alpha)

In effect, the equation for single exponential smoothing states that the new forecast is equal to the previous forecast plus "α" times the error of the previous forecast. (For the initial forecast on a series of data points, two prior year actual figures are required; S'_t is set to X_{t-1}.) The value of the weighting factor (α) must be between zero and one. As the value of alpha approaches 1, the new forecast will include an increasing adjustment for the error of the preceding forecast. However, in a situation of steady growth or decline even with α = 1, single exponential smoothing will consistently under or over-project. For a demonstration of this fact see Wheelwright and Makridakis (1973, p. 46). This deficiency is particularly critical in enrollment forecasting at the institutional level, where the principle focus most generally is on next year's forecast as opposed to long-term projections.

Whereas single exponential smoothing consistently under or over-predicts in a trend situation, double exponential smoothing adjusts well to trend variations. Essentially a repeated or "double" application of the single exponential smoothing formula, it provides greatly improved results. The basic procedures of the double exponential smoothing process involve three steps:

1. Use single exponential smoothing to establish an initial forecast (see formula above).
2. Apply an additional exponential smoothing iteration to the result of Step 1 for a second level forecast as follows:

$$S''_{t+1} = \alpha S'_{t+1} + (1 - \alpha) S''_t$$

Where S''_{t+1} = Second level double exponential forecast for period $t + 1$

S''_t = Second level double exponential forecast for period t

3. Adjust the original single exponential forecast by the difference between itself and the forecast of the second iteration to allow for trend variation as follows:

$$A = 2S'_{t+1} - S''_{t+1}$$

$$B = \frac{\alpha}{1-\alpha}(S'_{t+1} - S''_{t+1})$$

$$S_{t+m} = A + BM$$

Where S_{t+m} = Third level double exponential forecast

m = Periods ahead that are to be forecasted.

It is this third level forecast that should be taken as the final estimate.

Appropriate Weight Selection

A major drawback to the application of double exponential smoothing in practice is the question of how to determine an appropriate value of the weighting factor, or alpha (Wheelwright and Makridakis, 1973, p. 50). Literature on the subject typically suggests that the weighting factor should be chosen high (.8 or .9) to be highly responsive to recent changes in the underlying pattern and low (.1 or .2) to provide less sensitive response to current data fluctuation, as for example in Starr (1971), and Hillier and Lieberman (1974). The variation in effect of the selected weighting factor can be substantial. Table 1 is a set of actual headcount enrollment figures for a large public university for the years 1968 through 1978, while Table 2 presents the range of projections that would result from using these data and double exponential smoothing to predict the 1978 Fall enrollment with nine different alpha values.

Table 1

Headcount Enrollment Data

Fall Semester	Enrollment	Increase Over Preceding Year N	Increase Over Preceding Year %
1968	23,341		
1969	25,139	1,798	7.7
1970	26,425	1,286	5.1
1971	26,564	139	0.5
1972	27,322	758	2.9
1973	28,724	1,402	5.1
1974	31,021	2,297	8.0
1975	32,423[a]	1,402	5.1
1976	34,366	1,943	6.0
1977	35,278	912	2.7
1978	36,705[a]	1,427	4.0

[a] Differ from official reported figures due to definition changes. Figures adjusted to ensure comparability.

Table 2

Result of Using Nine Different Alpha Values
to Estimate Fall 1978 Enrollment

Alpha	Projection	Error From Actual (36,705) N	%
.9	36,383	- 322	0.9
.8	36,558	- 147	0.4
.7	36,712	+ 7	0.02
.6	36,826	+ 121	0.3
.5	36,849	+ 144	0.4
.4	36,687	- 18	0.05
.3	36,128	- 577	1.6
.2	34,683	-2022	5.5
.1	31,227	-5478	14.9

As illustrated in Table 2, the "heavily smoothed" alpha values of .2 and .1 would have resulted in projection errors of 5.5% and 14.9% respectively. The "highly responsive" values of .9 or .8 would have resulted in projection errors of less than 1%, but were not nearly as accurate as the alpha value .7 which came within 7 of the actual enrollment or .4 which came within 18. The literature mentioned earlier does not include an objective method which would have led to such a subtle choice of weight factor values as .7 or .4.

Two Alternatives

Two methods for objectively selecting the "best" weight factor for projecting future enrollments using double exponential smoothing have been tested. (I am especially indebted to John D. Parker and Joseph J. Matt of the Office of Institutional Studies and Planning at Arizona State University for ideas and collaboration in the early development of these procedures.) Based on the concept of "retrospective simulation" suggested by Hillier and Lieberman (1974, p. 524) the methods may be implemented using either a computer or an extended capacity calculator. A third method has been tested by the author which first analyzes historic data for cyclic patterns and then applies one of three smoothing formulas using a range of weight factors selected by the user. Because of the additional elements, relative complexity and hardware/software - specific nature of this approach (a Honeywell "TCAST" interactive FORTRAN program) it will not be discussed here. A significant advantage of the methods that follow is their relative simplicity.

The first of these methods has been called the "Looping Test of Alpha" and is based on the assumption that college enrollments have an inherent measure of stability due to the continuing student base. In this light, the method holds that pattern-based projections of changes in enrollment levels should be most sensitive to recent experience because enrollments in the

immediate future will be most strongly influenced by the current base. Therefore, a comparison at the last, or most recent actual data point, can be effectively used to guide in the choice of a weighting factor for the following year's projection.

Procedurally, a "looping" programmable calculator program was used to test the ability of each of nine values of alpha (α = .1 through .9) to predict the latest known value in a series of data points. The information derived was then used to estimate the expected value of the next (unknown) value.

In the example of the data provided above, double exponential smoothing with an alpha of .4 would come closest to Fall 1977 with a projection of 35,364 as compared to the 35,278 actual. As shown in Table 2, if the alpha factor of .4 were then used to estimate Fall 1978, it would have produced a highly acceptable result, estimating the enrollment at 36,687, just 18 under the actual.

To some extent, the degree of accuracy in this example is almost certainly coincidental. In fact, the Looping Test of Alpha method tends to over-react where there are relatively wide fluctuations from year to year, as in the sample inventory data provided by Wheelwright and Makridakis (1973, p. 45). When the underlying pattern exhibits this type of characteristic, conventional wisdom would dictate that a "highly smoothing" value of .2 should be selected, as in fact Wheelwright and Makridakis did in their example. But why .2? Why not .3 or .1?

The second method tested was designed to answer this kind of question, given a series of data points exhibiting relatively wide fluctuations around a general trend line. In light of the sharp short term fluctuations, the Looping Test of Alpha method would likely produce unacceptable results as explained earlier. However, the general trend characteristic of the data makes simpler methods such as single exponential smoothing or simple averages also

inappropriate as these tend to consistently under-predict or over-predict in a trend situation. To overcome these drawbacks while retaining the basic advantages of double exponential smoothing, an "Average Absolute Difference" method was designed and tested.

With this method, a programmable calculator was programmed to compare predicted vs. actual values of each of nine values of alpha (α = .1 through .9) over a series of data points. In this case, first the average absolute difference between the predicted and actual values over the complete series was computed. This information was then used to facilitate selection of the alpha value with the smallest average absolute error for use in estimating future values.

To return to the example enrollment data provided in Table 1, the alpha value with the lowest average absolute difference between actual and projected values through 1977 would be .7 (av. = 614). As illustrated in Table 2, if double exponential smoothing with alpha = .7 were then used to forecast Fall 1978 it would have produced a highly accurate forecast of 36,712. (Interestingly, this method of weight factor selection applied to the Wheelwright and Makridakis data reveals that alpha = .3 has the lowest average absloute difference over 23 data points, edging out .2 by a narrow margin. In this case, the conventional wisdom of selecting a highly smoothed value for relatively unstable data is upheld, but an objective method is provided for making a subtle improvement in choice.)

Summary and Conclusions

Because of the abundance and inherent complexity of factors influencing college and university enrollments, pattern-based forecasting must always be used with caution. These methods start with the generally erroneous assumption (in an exact sense) that the observed pattern will continue into the

future. Pinpoint accuracy is not to be expected and, when it occurs, should be viewed as coincidental and suspect. Severe impending discontinuities in the underlying pattern will not be anticipated by any pattern-based formula. However, in spite of these basic shortcomings, pattern-based forecasting will continue to have utility for providing baseline estimates and bracketing the figures generated by other techniques.

Double exponential smoothing is an attractive pattern-based method which is relatively simple, easy to apply and adaptable to a variety of circumstances. In most cases, weight factor selection has been viewed as an essentially subjective process based on visual inspection of the historical data. This has generally precluded the probability of making subtle choices in alpha values that might improve forecast accuracy. Furthermore, the conventional recommendation that higher alpha values are most sensitive to recent changes in the pattern while lower values have a greater smoothing effect, might mislead some persons into thinking that step-wise changes in alpha selection will have a step-wise effect on level of error. As illustrated in Table 2, the weight factor effect on error is not linear but parabolic, with two opposing alpha values producing similar projections. Where this occurs, it is possible with both of the weight selection methods described above to examine the slope (projected direction and rate of change; "B" in the formula provided earlier) to determine which of two very close alpha projections conforms most closely to recent rate of change experience.

The often critical importance of enrollment forecasts dictates that it is desirable to examine the results of as many different methods as is practical before arriving at a final prediction. Both of the techniques described in this article can effectively be used to supplement other methods in producing reasonable mathematical estimates of enrollment.

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